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ON A CERTAIN SOLUTION OF THE PROBLEM OF MOTION OF A GYROSCOPE ON GIMBALS

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The equations of motion of a heavy gyroscope on gimbals are integrated for an arbitrary position of the center of mass of the gyro housing.

In 1958 Chetaev [1] investigated the motion of a heavy gyro on gimbals in the case of vertical position of the outer gimbal axis of rotation (output axis). The center of gravity of the housing and gyro was assumed to coincide with the axis of symmetry of the gyro. Chetaev reduced the problem of integrating the equations of motion to quadratures. These quadratures can be readily extended to the case where the gyroscope is acted along the axis of rotation of its housing by a moment of external forces which is an arbitrary integrable function of the angle of nutation.

This problem was considered in [2] under certain assumptions concerning the moments of inertia of the system elements and for certain specific initial data.

1. Let us consider a gyro on gimbals under the assumption that the fixed axis of rotation of the outer gimbal is in vertical position. We introduce two right-handed coordinate systems with a common origin at a fixed point O of the gyroscope. The axis ξ_3 of the fixed coordinate system ξ_1, ξ_2, ξ_3 is directed vertically upward along the axis of rotation of the outer gimbal; the axes ξ_1 and ξ_2 lie in the horizontal plane. The axes η_1 and η_2 of the moving coordinate system $\eta_1\eta_2\eta_3$ (which is rigidly attached to the gyroscope housing) are directed along the axis of rotation of the housing and along the axis of symmetry of the gyro, respectively. The position of the system under consideration in the space $\xi_1\xi_2\xi_3$ is defined by the three Euler angles, namely the angle of precession ψ, the angle of nutation θ, and the angle of proper rotation φ of the gyro relative to the coordinate system $\eta_1\eta_2\eta_3$. Let $\eta_1\eta_2\eta_3$ be the principal axes of inertia of both the gyro and its housing; let $A_1 = A_2$, A_3 be the principal moments of inertia of the gyro, and let B_1, B_2, B_3 be the principal moments of inertia of the housing; we denote the moment of inertia of the outer gimbal relative to the axis ξ_3 by C_3.

The kinetic energy of the system is given by the expression

$$2T = \psi^{\cdot 2} (A_0 - C_0 \cos^2\theta) + \theta^{\cdot 2} B_0 + A_3 (\varphi^{\cdot} + \psi^{\cdot} \cos\theta)^2$$

$$A_0 = C_3 + B_2 + A_1, \quad B_0 = A_1 + B_1, \quad C_0 = B_2 + A_1 - B_3$$

The dot superscripts denote derivatives with respect to time. From now on we assume that $C_\theta = 0$ [3]. The coordinates of the center of gravity of the housing will be denoted by $\eta_{10}\eta_{20}\eta_{30}$, and the weight by P_1. The corresponding gyro parameters have the values $0, 0, l, P$. The force function is

$$U = -P_1\eta_{20}\sin\theta - (P_1\eta_{30} + Pl)\cos\theta$$

Let us assume that the system under consideration is acted on by weights only. Then the equation of motion of the system written in the Lagrange form of the second kind,

$$[A_0\psi^{\cdot} + A_3 \cos\theta\, (\varphi^{\cdot} + \psi^{\cdot}\cos\theta)]^{\cdot} = 0$$

$$B_0\theta^{\cdot\cdot} + A_3\psi^{\cdot} \sin\theta\, (\varphi^{\cdot} + \psi^{\cdot}\cos\theta) = -P_1\eta_{20}\cos\theta + G\sin\theta \tag{1.1}$$

$$A_3 (\varphi^{\cdot} + \psi^{\cdot}\cos\theta)^{\cdot} = 0, \quad G = P_1\eta_{30} + Pl$$

has the first integrals

$$A_3 (\varphi^{\cdot} + \psi^{\cdot}\cos\theta) = H$$

$$A_0\psi^{\cdot} + A_3 (\varphi^{\cdot} + \psi^{\cdot}\cos\theta)\cos\theta = L \tag{1.2}$$

$$A_0\psi^{\cdot 2} + B_0\theta^{\cdot 2} + 2P_1\eta_{20}\sin\theta + 2G\cos\theta = M$$

Here H, L and M are integration constants.

2. System (1.2) yields the following system of differential equations for the Euler angles $\psi, \theta\ \varphi$:

$$\psi^{\cdot} = \psi_0^{\cdot} + (\cos\theta_0 - \cos\theta)\, H / A_0 \tag{2.1}$$

$$\sqrt{A_0B_0}\,\theta^{\cdot} = [\theta_0^{\cdot 2} A_0 B_0 + 2A_0P_1\eta_{20} (\sin\theta_0 - \sin\theta) + 2A_0 (G - H\psi_0^{\cdot}) (\cos\theta_0 - \cos\theta) - H^2 (\cos\theta_0 - \cos\theta)^2]^{1/2} \tag{2.2}$$

$$\varphi^{\cdot} = H / A_3 - \cos\theta\, [(\cos\theta_0 - \cos\theta)\, H / A_0 + \psi_0^{\cdot}] \tag{2.3}$$

The plus sign in the right side of (2.2) follows from the assumption that $\theta(t)$ does not decreases for $t = t_0$; our choice is dictated solely by the wish to be specific, and this restriction will be lifted below.

Let us change variables in Eq. (2.2) by setting

$$x^{-1} = \mathrm{tg}\, {}^1\!/_2(\theta - \theta_0)$$

As a result we obtain

$$-2\sqrt{A_0B_0}\,x^{\cdot} = (b_0x^4 + 4b_1x^3 + 6b_2x^2 + 4b_3x + b_4)^{1/2}$$

$$b_0 = A_0B_0\theta_0^{\cdot 2}, \quad b_1 = A_0 (G - H\psi_0^{\cdot}) \sin\theta_0 - A_0P_1\eta_{20}\cos\theta_0$$

$$3b_2 = A_0B_0\theta_0^{\cdot 2} + 2A_0 (G - H\psi_0^{\cdot})\cos\theta_0 + 2A_0P_1\eta_{20}\sin\theta_0 - 2H^2\sin^2\theta_0$$

$$b_3 = A_0 (G - H\psi_0^{\cdot}) \sin\theta_0 - A_0P_1\eta_{20}\cos\theta_0 - H^2\sin 2\theta_0$$

$$b_4 = A_0B_0\theta_0^{\cdot 2} + 4A_0 (G - H\psi_0^{\cdot})\cos\theta_0 + 4A_0P_1\eta_{20}\sin\theta_0 - 4H^2\cos^2\theta_0$$

Setting $x = y - b_1 / b_0$, we obtain

$$-2\sqrt{A_0B_0/b_0}\,y^\cdot = \sqrt{y^4+6c_2y^2+4c_3y+c_4}$$

$$c_2 = \frac{b_0b_2-b_1^2}{b_0^2}, \qquad c_3 = \frac{b_3b_0^2-3b_0b_1b_2+2b_1^3}{b_0^3}, \qquad c_4 = \frac{b_4b_0^3-4b_0^2b_1b_3+6b_0b_1^2b_2-3b_1^4}{b_0^4}$$

Let us introduce the new variable z by means of the formula [4]

$$-\sqrt{y^4+6c_2y^2+4c_3y+c_4} = y^2+c_2-2z$$

Squaring,

$$4c_2y^2+4c_3y+c_4+4y^2z+4c_2z-4z^2-c_2^2 = \qquad (2.4)$$

and differentiating, we obtain

$$\frac{dy}{y^2+c_2-2z} = \frac{dz}{2c_2y+2yz+c_3}$$

Let us solve (2.4) for y,

$$y = \frac{-c_3 \pm \sqrt{4z^3-g_2'z-g_3'}}{2(z+c_2)}, \qquad g_2' = c_4+3c_2^2, \quad g_3' = c_2c_4-c_3^2-c_2^3 \qquad (2.5)$$

Let us construct the Weierstrass elliptic function $\wp(w)$ with the invariants g_2' and g_3' and set $z = \wp(w)$. Rewriting (2.5) in the form

$$(2zy+2c_2y+c_3)^2 = 4z^3-g_2'z-g_3'$$

and recalling the following consequence of the definition of $\wp(w)$:

$$\wp'(\omega) = -\sqrt{4\wp^3(\omega)-g_2'\wp(\omega)-g_3'}$$

we conclude that

$$-\frac{dy}{\sqrt{y^4+6c_2y^2+4c_3y+c_4}} = \frac{dy}{y^2+c_2-2z} = -\frac{dz}{2c_2y+2yz+c_3} = dw$$

Hence,

$$w = {}^1/_2\sqrt{b_0/A_0B_0}\,(t-t_0)$$

Converting back to the original variables and applying the homogeneity formula to the functions $\wp(w)$ and $\wp'(w)$ we obtain

$$\theta = \theta_0 + 2\,\mathrm{Arc\,tg}\,\frac{2[b_0\wp(\tau)+b_0b_2-b_1^2]}{+\sqrt{b_0}\,\wp'(\tau)-2b_1\wp(\tau)-b_0b_3+b_1b_2} \qquad (2.6)$$

The functions $\wp(\tau)$ and $\wp'(\tau)$ must be computed for the invariants

$$g_2 = b_0b_4-4b_1b_3+3b_2^2, \qquad \tau = (t-t_0)/2\sqrt{A_0B_0}$$
$$g_3 = b_0b_2b_4-b_1^2b_4-b_2^3-b_0b_3^2+2b_1b_2b_3$$

The plus or minus sign in the denominator of (2.6) must be chosen according to whether $\theta(t)$ decreases or increases for $t = t_0$. The above analysis becomes methodologically invalid for $b_0 = 0$, but in this case the expression for the function $\theta(t)$ can be obtained from (2.6) by taking the limit as $b_0 \to 0$ by virtue of its continuity with respect to this parameter. The resulting function

$$\theta = \theta_0 + 2\,\mathrm{Arc\,tg}\,\frac{2b_1}{2\wp(\tau)-b_2} \qquad (2.7)$$

is the solution of (2.2). The variables ψ and φ can therefore be found by quadratures in the form of explicit functions of the time t.

3. Let us consider some properties of the above solution. In the event that

$$\theta_0^\cdot = 0, \quad (G-H\psi_0^\cdot)\sin\theta_0 = P_1\eta_{20}\cos\theta_0 \qquad (3.1)$$

we have the stationary solutions

$$\theta \equiv \theta_0\ \theta^\cdot \equiv 0,\ \psi^\cdot \equiv \psi_0^\cdot,\ \varphi^\cdot \equiv \varphi_0^\cdot \qquad (3.2)$$

The Routh-altered force function

$$U_1(\theta) = -P_1\eta_{20}\sin\theta - G\cos\theta - (L - H\cos\theta)^2/2A_0$$

has a maximum at the equilibrium position, provided that

$$U_1''(\theta_0) = P_1\eta_{20}\sin\theta_0 + (G - H\psi_0^{\cdot})\cos\theta_0 - H^2\sin^2\theta_0/A_0 < 0 \qquad (3.3)$$

Since the first integrals are continuous [5], it follows that the sufficient condition of unconditional stability is the stability of the stationary solutions with respect to variables (3.2).

In the case

$$U_1''(\theta_0) > 0$$

the force function has a minimum in the equilibrium position, and this is determined by the lowest-order terms in the expansion of this function, which (by virtue of Liapunov's theorem [6]) is the sufficient condition of instability of the stationary solutions with respect to the variables θ and $\theta^{\cdot}$.

In the case

$$\Delta = P_1^2\eta_{20}^2 + (G - H\psi_0^{\cdot})^2 \neq 0$$

we can make use of (3.1) and rewrite (3.3) as

$$U_1''(\theta_0) = \pm\sqrt{\Delta} - H^2P_1^2\eta_{20}^2/A_0\Delta$$

Here the plus or minus sign is chosen in accordance with the signs of the quantities $P_1\eta_{20}$, $G - H\psi_0^{\cdot}$ and according to the position of θ_0 in the axis θ.

At the equilibrium positions $\theta_0 \pm \pi$ next to θ_0 the sign changes from the original sign; this means that every equilibrium position is stable for

$$A_0\,|\Delta^{3/2}| < H^2P_1^2\eta_{20}^2$$

while for the opposite inequality the stable equilibrium positions are separated by unstable ones along the θ-axis; this coincides with the familiar [7, 8] conditions of stability of rotation of a gyro about the vertical.

If $U_1''(\theta_0) = 0$ and if the first nonzero derivative is of even order, then U_1 has an extremum at the equilibrium position, and the question of stability is resolved by the two theorems cited above.

When $U_1''(\theta_0) = 0$ and when the first nonzero derivative is of uneven order the force function U_1 does not have an extremum at the equilibrium position, and stationary solution (3.2) is unstable with respect to the variables θ and $\theta^{\cdot}$; this follows from Chetaev's theorem [9] on the instability of equilibrium in the case where the force function is analytic and does not have an extremum at the equilibrium position.

However, for systems with one degree of freedom equilibrium is also unstable for less rigid restrictions on the force function than those imposed in [9].

4. Let us consider a mechanical system with one degree of freedom whose equation of motion can be written as

$$\theta^{\cdot\cdot} = \frac{dU_1(\theta)}{d\theta} \qquad (4.1)$$

We assume that the equilibrium positions are known and that they lie at finite distances from the equilibrium position (0, 0) under investigation. We also assume that the force function $U_1(\theta) \not\equiv 0$ is continuously differentiable and that it does not have an extremum at the equilibrium position; moreover, $U_1(0) = 0$.

Let us construct in the plane $\theta\theta^{\cdot}$ a circle of finite radius (not larger than the distance of the nearest equilibrium position) with its center at the origin. Let us consider the

function
$$V = \theta^{\cdot} U_1(\theta)$$
which is positive inside the sector of the circle
$$\theta^{\cdot} > 0, \qquad U_1(\theta) > 0 \qquad (4.2)$$
and vanishes at the rays bounding this sector. By virtue of the absence of an extremum at zero and the conditions $U_1(0) = 0$, $U_1(\theta) \not\equiv 0$ the function $U_1(\theta)$ necessarily assumes positive values near the equilibrium position. The function $U_1(\theta)$ cannot vanish inside sector (4.2), since in this case (by virtue of Rolle's theorem) we would have $dU_1(\theta)/d\theta = 0$ inside the sector, which cannot happen by virtue of our choice of the radius of the circle. Hence, sector (4.2) always exists and is the domain $V > 0$. The derivative
$$V^{\cdot} = \frac{dU_1(\theta)}{d\theta}\,[U_1(\theta) + \theta^{\cdot 2}]$$
constructed by virtue of the equations of perturbed motion (4.1) is positive in the domain $V > 0$, since $U_1(\theta) > 0$ in this domain, and the derivative $dU_1(\theta)/d\theta$, being a continuous function positive in the neighborhood of zero, perserves its sign all the way to the nearest equilibrium position, where it vanishes. Thus, the function V satisfies the conditions of Chetaev's theorem on instability [9].

5. We note that the authors of [2] obtained results which do not agree with those above in their common domain of definition. Thus, taking the initial data in the form
$$\theta_0 = {}^1/_2\pi, \qquad \theta_0^{\cdot} = 0, \qquad G - H\psi_0^{\cdot} = 0$$
and following the recommendations of Sect. 5 in [2], we obtain
$$\cos\theta = -\cos\frac{Ht}{\sqrt{A_0B_0}}, \qquad \psi^{\cdot} = \psi_0^{\cdot} + \frac{H}{A_0}\,|\cos\frac{Ht}{\sqrt{A_0B_0}}$$
On the other hand, our expressions at the same initial data and under the condition $P_1\eta_{20} = 0$ (which ensures the coincidence of the models) yield
$$\theta \equiv {}^1/_2\pi, \qquad \psi^{\cdot} \equiv \psi_0^{\cdot}$$
This is apparently due to the fact that the authors of [2] (Sect. 5) assume that the function
$$u = \mathrm{sn}\,[\tau - K(u_1)]$$
is the solution of the equation
$$du / d\tau = \sqrt{(u - u_1)(u_2 - u)(1 - u^2)}$$
for $u_1 + u_2 = 0$ and the initial condition $u(0) = u_1$. However, the true solution is
$$u = u_1\,\mathrm{sn}\,[\tau + K(u_1)]$$

6. Let us extend our consideration of the properties of the function $\theta(t)$, setting $b_0 = 0$, $b_1 \neq 0$. As it varies along the real axis the function $\wp(\tau)$ assumes all values from $+\infty$ to e, the latter being the largest root of the polynomial
$$F(z) = 4z^3 - g_2 z - g_3 \qquad (6.1)$$
The properties of the function $\theta(t)$ in the case in question depend on the ratio of ${}^1/_2b_2$ to e and on the multiplicity of the latter. It is conveneient to break down analysis of the situation into three cases, since
$$F({}^1/_2b_2) = b_1^2 b_4$$
1°. Let $b_4 < 0$. Then
$$^1/_2b_2 < e$$

2°. Let $b_4 = 0$. Then

$$z_1 = {}^1/_2 b_2, \qquad z_{2,3} = {}^1/_4 \left(-b_2 \pm \sqrt{9b_2^2 - 16b_1b_3}\right)$$

are the roots of (6.1).

a) If $b_2 \geqslant 0$, then

$$e = z_2 \quad \text{for} \quad b_1b_3 \leqslant 0$$
$$e = z_1 \quad \text{for} \quad b_1b_3 > 0$$

b) If $b_2 < 0$, then

$$e = z_2 \quad \text{for} \quad \Delta = 9b_2^2 - 16b_1b_3 \geqslant 0$$
$$e = z_1 \quad \text{for} \quad \Delta < 0$$

3°. Let $b_4 > 0$. Then

$$e < {}^1/_2 b_2 \quad \text{for} \quad g_2^3 - 27g_3^2 < 0 \quad \text{or} \quad {}^1/_3 g_2 \sqrt{{}^1/_3 g_2} = g_3$$

In the case $g_2^3 - 27g_3^2 \geqslant 0$ the function $F(z)$ has three real roots, and the sufficient conditions for ${}^1/_2 b_2$ being the upper boundary of the real roots of $F(z)$ become the necessary conditions and assume the form $b_4 > 0$, $b_1b_3 > 0$, $b_2 > 0$.

In all cases $e = \sqrt{{}^1/_{12} g_2}$ is the largest multiple root of $F(z)$ for ${}^1/_3 g_2 \sqrt{{}^1/_3 g_2} = -g_3$.

We have therefore obtained an algorithm which enables us to determine the ratio of ${}^1/_2 b_2$ to e and the multiplicity of the latter. Analysis of (2.7) enables us to formulate the following properties of $\theta(t)$:

a) $\theta_0 \leqslant \theta \leqslant \theta_1$, $\theta_1 = \theta_0 + 2\,\mathrm{Arctg}\,[2b_1 / (2e - b_2)]$

b) If $e > {}^1/_2 b_2$, then $|\theta_1 - \theta_0| < \pi$

c) If $e = {}^1/_2 b_2$, then $|\theta_1 - \theta_0| = \pi$

d) If $e < {}^1/_2 b_2$, then $\pi < |\theta_1 - \theta_0| < 2\pi$

e) If e is a multiple root of $F(z)$, then $\theta(t)$ reaches the value θ_1 after infinite time.

The assumption $b_0 = 0$ by virtue of which the properties of $\theta(t)$, were considered means that θ_0 is a root of the right side of (2.2).

If θ_0 is not a root ($b_0 \neq 0$), but if simple roots do exist, then, having found one of these roots θ_{01} and taking it to be the initial value of $\theta(t)$ for the previous H, L and M, we obtain a solution equivalent in its properties to that considered above ($b_0 = 0$).

If $b_0 \neq 0$ and if the right side of (2.2) has no roots, then $\theta(t)$ is a monotonic function which increases by 2π in the period $\wp(\tau)$. On the other hand, if $b_0 \neq 0$ and if the right side of (2.2) has multiple roots only, then consideration of the properties of the solution to the case $b_0 = 0$ by the above method is impossible, since in this case investigation of the initial solution must be replaced by investigation of the stationary solution. In this case $\theta(t)$ approaches a multiple root asymptotically.

Summarizing the above, we can say that solutions which are stationary, monotonic, asymptotic, and periodic with respect to the nutation angle θ are possible.

Following the idea of the authors of [10], who were the first to obtain an exact formula for the drift of a balanced gyroscope (an exact formula for the drift of a balanced gyroscope was later proposed in [11], the average drift rate in the last case reducible to $b_0 = 0$ is given by

$$\langle \dot\psi \rangle = \left[\frac{1}{A_0} \int_{\theta_0}^{\theta_1} \frac{L - H\cos\theta}{\sqrt{f(\theta)}}\, d\theta\right] \left[\int_{\theta_0}^{\theta_1} \frac{d\theta}{\sqrt{f(\theta)}}\right]^{-1}$$

Here $f(\theta)$ is the radicand on the right side of (2.2), and θ_0 and θ_1 are the roots.

If the conditions

$$A_0 (G - H\psi_0^{\cdot}) - H^2 \cos\theta_0 = A_0 G - LH = 0 \qquad (6.2)$$

are fulfilled, we have

$$f(\theta) = 2A_0 P_1 \eta_{20} (\sin\theta_0 - \sin\theta) + H^2 (\cos^2\theta_0 - \cos^2\theta)$$

which is an even function if either of the points $\pm \frac{1}{2}\pi$ is taken as the new origin on the axis θ, since $f(\theta)$ depends solely on $\sin\theta$. Once a new origin has been chosen, $\cos\theta$ becomes an odd function, which implies that

$$\int_{\theta_0}^{\theta_1} \frac{\cos\theta \, d\theta}{\sqrt{f(\theta)}} = 0$$

since it is an integral of an odd function over an interval having the origin as its mid-point. The average drift rate when (6.2) is fulfilled becomes particularly simple,

$$\langle \psi^{\cdot} \rangle = \frac{L}{A_0} = \psi_0 + \frac{H \cos\theta_0}{A_0}$$

The fact of zero drift of a balanced gyroscope for $L = 0$ was pointed out in [11] and later rediscovered by the authors of [2] in the case of a balanced gyro with restrictions imposed on the moments of inertia ($C_0 = 0$). We note that in the latter case fulfillment of the condition $G = L = 0$ ensures that $\langle \psi^{\circ} \rangle = 0$ regardless of whether $P_1\eta_{20}$ is equal to zero.

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